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A UNIFIED ANALYSIS OF TRANSMISSION LINE DISCRIMINATORS FOR F.M. NOISE MEASUREMENTS

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Summary

The transmission line discriminator is a well known system for measuring phase noise of an oscillator. Using several forms of these circuits, (particularly those patented by Ashley et al.) [5,6] as a basis, we derive equations for the mixer I.F. output voltage, use these to determine the sensitivity of the discriminator and system threshold.

This is related to a minimum detectable frequency deviation. Combining these theoretical equations and computer simulations with experimental results, we provide a unified analysis of frequency discriminators which goes well beyond any analysis available in current literature.

Introduction

The use of the transmission line as delay line or resonator in F.M. discriminators has been well established in past literature. [2,5,6] What hasn't been accomplished up until this time is a unified analysis from the source input to the I.F. output. Within this paper we shall describe in such a manner several discriminators and show computer simulations which match closely with laboratory data.

Development of Output Voltage Equation

Referring to Fig 1, we begin our theoretical baseline with simple transmission line theory. For analysis purposes we will model the transmission line resonator as a shorted line with a stub tuner. From the transmission line equations, we can easily write the reflection coefficient of the tuned line.

$$(1)\quad \Gamma x = \frac{3\Gamma e^{j(\sigma+\beta)} - e^{-j(\sigma+\beta)} - e^{j(\sigma-\beta)} - \Gamma e^{-j(\sigma-\beta)}}{3\, e^{-j(\sigma+\beta)} - \Gamma e^{j(\sigma+\beta)} - e^{j(\sigma-\beta)} - \Gamma e^{-j(\sigma-\beta)}}$$

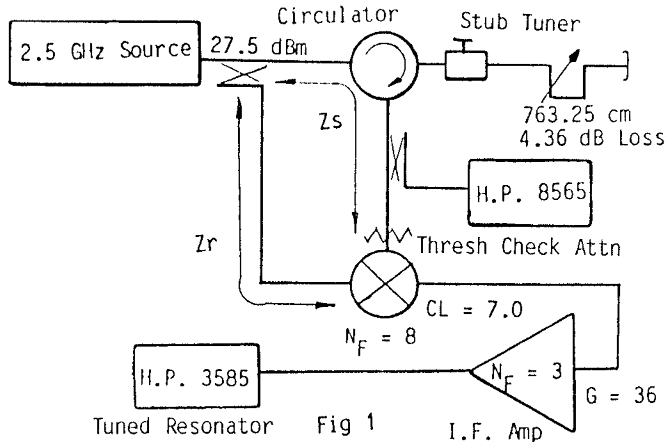
The signal applied to the mixer R.F. port is now expressible as:

$$(2)\quad Vs = |V|\, e^{j(\omega t+Bzs)}\, \Gamma x$$

Mixing with the L.O. signal of Cos (ωt-Zr) we finally come to an expression which describes the output of our mixer:

$$(3)\quad Vo = \frac{V}{2}\operatorname{Cos}(Z) - |V|\, \frac{-3\Gamma \operatorname{Cos}(Z-2\sigma-2\beta) - \Gamma\operatorname{Cos}(Z+2\sigma+2\beta) + \Gamma\operatorname{Cos}(Z-2\beta) - \Gamma\operatorname{Cos}(Z+2\beta) + \Gamma^2\operatorname{Cos}(Z-2\sigma) - \operatorname{Cos}(Z+2\sigma) + (\Gamma^2+3)\operatorname{Cos}(Z)}{-3\Gamma\operatorname{Cos}2(\sigma+\beta) + \Gamma\operatorname{Cos}2(\sigma-\beta) + (\Gamma^2-3)\operatorname{Cos}2\sigma - 2\Gamma\operatorname{Cos}2\beta + 5 + \Gamma^2}$$

Where:
- $|V|$ = Voltage Applied
- Γ = Loss in the Resonator
- σ = $2\pi S/\lambda$
- Z = -Zr+Zs
- Zr = $2\pi\, zr/\lambda$
- Zs = $2\pi\, zs/\lambda$
- β = $2\pi\, \ell/\lambda$
- ℓ = Line Length
- S = Stub Length



Tuned Resonator Discriminator

Fig 1

System Threshold

The threshold of our system depends upon the available noise power from the system. [1,2] This noise power can usually be isolated to the mixer and I.F. amplifier and can be calculated from the noise figures of these devices. This available noise voltage can be expressed as:

$$(4)\quad En = (50\times10^{-3})^{1/2}\left[10^{(N1-174-CL)/20} + (10^{N2/10}-1)^{1/2}\, 10^{-8.7}\right]10^{G1/20}$$

Where:
- CL = Conversion Loss of Mixer
- N1 = Noise Figure of Mixer.
- N2 = Noise Figure of Amplifier
- G1 - Gain of Amplifier

This equation is a good approximation out above 100KHz but if we look into diode noise characteristics we will see that it is rather poor below this frequency. [3,4] Below 100KHz we must deal with flicker noise. A useful simplification for our simulation purposes is to describe the flicker region as a 1/f increase in available noise power from the mixer. The noise voltage equation in this region can be expressed.

$$(5)\quad En_1 = 10^{G1/20}\left[(K/(F-Fc))^{1/2} + (50\times10^{-3})^{1/2}\,(10^{N2/10}-1)^{1/2}\; 10^{-8.7}\right]$$

Where:
- K = Arbitrary Constant
- F = Operating Frequency
- Fc = Center Frequency of Discriminator.

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Charactorization of the Discriminator

The minimum detectable delta f of the system can now be expressed as:

$$(6)\quad S = \frac{E_n}{dV_o/df}$$

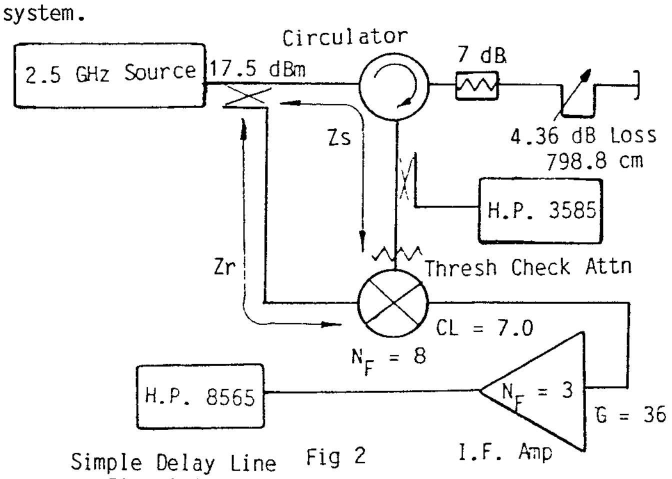
where dvo/df is obviously the slope of the discriminator as was discussed by Ashley et. al.[1,2] This equation gives us a unique indication of the minimum detectable delta f which has been our goal.

Alternate Topologies

This analysis can easily be extended to alternate forms of transmission line discriminators. Another familiar form is that of Fig 2. The output voltage equation for this configuration is:

$$(7)\quad V_o = |V|\ r\ 10^{G1/20} \cos(Z+2B)$$

Using this and Equation 7 we can calculate again the minimum detectable delta f for our discriminator system.



Simple Delay Line Discriminator Fig 2

Computer Simulation and Laboratory Results

For the purpose of computer simulation we have chosen the H.P. 87 because of it's excellent graphics capabilities. From the program listed we produced the final plots Figures 3 and 4. These plots were run as direct simulation of circuits which were set up in the Laboratory. The circuits tested and simulated were as pictured in Figures 1 and 2. The data shown in Table 1 for the threshold level and discriminator slope were taken in accordance with procedures described by Ashley et. al.[1,2] If we return to Figures 3 and 4 we see that the laboratory data, which have been plotted as x's on the computer simulation plots, are in excellent agreement with that calculated from circuit parameters. The differences are well within the approximations of circuit parameters. We have thus shown that by starting with known circuit parameters we can accurately predict the response of our discriminator circuits.

References

1. Ashley et. al. "Measurement of Oscillator Noise at Microwave Frequencies" MTT-16 '68
2. Ashley et. al. "Measurement of Noise in Microwave Transmitters" MTT-25 '77
3. H.P. APP NOTE 956-3 "Flicker Noise In Shottky Diodes"
4. Microwave Associate Bulletin 4251
5. Barley et. al. Patent #40002.969
6. Ashley et. al. Patent #40002.970

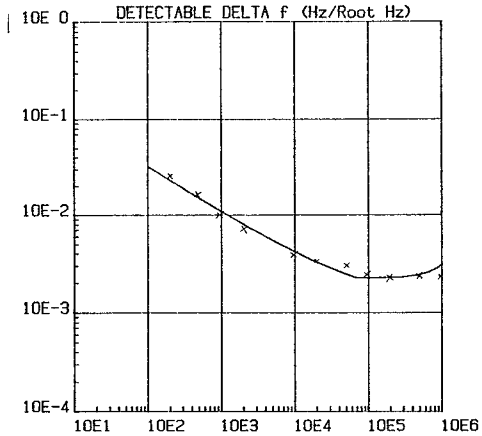


FIGURE 3

MINIMUM DETECTABLE Δf
FOR DISCRIMINATOR OF FIG 1

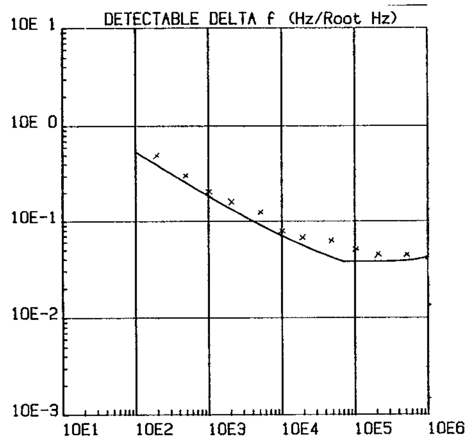


FIGURE 4

MINIMUM DETECTABLE Δf
FOR DISCRIMINATOR OF FIG 2

			FIG 1	FIG 2
			S=22.4K	S=454K
Freq	dBm	RBW	Δ f	Δ f
200	-91.5	30	.024	.495
500	-96	30	.0145	.293
1K	-99	30	.0103	.208
2K	-101.7	30	.0075	.153
5K	-104.1	30	.0055	.115
10K	-102.4	100	.0038	.077
20K	-103.5	100	.0033	.068
50K	-104.3	100	.0030	.062
100K	-101.5	300	.0024	.049
200K	-102.2	300	.0022	.045
500K	-102.3	300	.0022	.045
1M	-102.7	300	.0021	.043

TABLE 1
LABORATORY DATA

```
10 ! THIS PGM IS SAVED AS MTTPGM
20 RAD
30 DIM A(2,600)
40 DISP "WHICH TYPE OF DISCRIMINATOR"
50 DISP "ENTER 1 FOR TUNED RES"
60 DISP "ENTER 2 FOR SIMPLE DELAY LINE"
70 INPUT C
80 DISP "INPUT LIMITS ON F (FLO,FHI) IN GHz"
90 INPUT F1,F3
100 DISP "SIGNAL PWR dBm"
110 INPUT P1
120 DISP "INPUT CENTER FREQ"
130 INPUT F2
140 DISP "INPUT STEP SIZE"
150 INPUT I1
160 DISP "INPUT SIGNAL PATH LENGTH"
170 INPUT S1
180 DISP "INPUT REF PATH LENGTH"
190 INPUT R1
200 DISP "INPUT MIXER CONVERSION LOSS"
210 INPUT C1
220 DISP "INPUT MIXER NF"
230 INPUT N8
240 DISP "IF AMP GAIN dB"
250 INPUT G1
260 DISP "CIRCULATOR/HYBRID LOSS"
270 INPUT U1
280 DISP "IF AMP NF"
290 INPUT N9
300 K2=-174
310 N7=K2-C1+N8
320 N6=10^(N7/10)*.001
330 E1=SQR (N6*50)
340 K1=E1^2*70000
350 V1=SQR (10^((P1-C1-U1)/10)*50/1000)
360 ON C GOSUB 1210,1420
370 !   THIS IS THE GRAPHICS ROUTINE
380 !  I.F. OUTPUT PLOT
390 PLOTTER IS 705
400 IF F3-F2<= .1 THEN X1=.01
410 IF F3-F2<= .01 THEN X1=.001
420 IF F3-F2<= .001 THEN X1=.0001
430 GRAPHICS
440 PEN 1
450 LORG 5
460 MOVE 60,3
470 LABEL "I.F. OUTPUT (VOLTS) VS FREQ (GHz)"
480 MOVE 105,90
490 LABEL USING "DD.D,7A" ; D1,"dB Attn."
500 LOCATE 15,115,7,95
510 SCALE F1,F3,-1,1
520 AXES X1,.1,F1,0,10,2,2
530 MOVE F1+X1,-.1
540 IF F2<= .1 THEN LABEL F1
550 IF F2<= 1 AND F2>.1 THEN LABEL F1
560 IF F2<= 10 AND F2>1 THEN LABEL F1
570 FOR X=F1+10*X1 TO F3 STEP 10*X1
580 MOVE X,-.1
590 IF F2<= .1 THEN LABEL X
600 IF F2<= 1 AND F2>.1 THEN LABEL X
610 IF F2<= 10 AND F2>1 THEN LABEL X
620 NEXT X
630 MOVE F3-X1,-.25
640 LABEL "F GHz"
650 FOR Y=-1 TO 1 STEP .2
660 LORG 8
670 MOVE F1-X1/2,Y
680 LABEL Y
690 NEXT Y
700 PEN 2
710 FOR N=0 TO T
720 PLOT A(0,N),A(1,N)
730 NEXT N
740 DISP "PLACE PAPER IN PLOTTER AND CONT"
750 PAUSE
760 !  THIS IS THE THRESHOLD PLOT
770 PLOTTER IS 705
780 PEN 1
790 H=ABS (A(2,1))
800 P3=LGT (H)
810 Y1=INT (P3+2)
820 H1=ABS (A(2,N1))
830 P4=LGT (H1)
840 Y2=INT (P4-1)
850 GRAPHICS
860 FRAME
870 MOVE 50,96
880 LABEL "DETECTABLE DELTA f (Hz/Root Hz)"
890 LOCATE 15,115,7,95
900 SCALE 1,7,Y2,Y1
910 AXES 1,1,1,Y2,10,10
920 E$="E"
925 X=1
930 FOR N=1 TO 7
940 LINE TYPE 1
950 MOVE N,Y2-.2
960 LABEL USING "DD,1A,D" ; 10,E$,N
963 MOVE X,Y1
967 DRAW X,Y2
970 FOR P=10^N TO 10^(N+1) STEP 10^N
980 X=LGT (P)
990 MOVE X,Y2+.05
1000 DRAW X,Y2
1010 NEXT P
1020 NEXT N
1030 FOR Q=Y2 TO Y1
1040 MOVE .3,Q
1050 LABEL USING "DD,1A,MD" ; 10,E$,Q
1053 MOVE 7,Y
1057 DRAW 1,Y
1060 FOR R=10^Q TO 10^(Q+1) STEP 10^Q
1070 Y=LGT (R)
1080 MOVE 1.05,Y
1090 DRAW 1,Y
1100 NEXT R
1110 NEXT Q
1120 PEN UP
1130 PEN 2
1140 FOR W=1 TO T-N1
1150 H=ABS (A(2,W))
1160 F9=(A(0,W+N1)-F2)*1000000000
1170 PLOT LGT (F9),LGT (H)
1180 NEXT W
1190 PEN UP
1200 END
1210 !  THIS IS THE TUNED RES. CALCS.
1220 DISP "INPUT ATTN dB"
1230 INPUT D1
1240 DISP "RES. LENGTH IN Cm"
1250 INPUT L1
1260 DISP "STUB LENGTH AS # OF WAVELENGTHS"
1270 INPUT S2
1280 S3=30000000000/(F2*1000000000*1.5)*S2
1290 GOSUB 1580
1300 RETURN
1310 G9=10^(-(D1/10))
1320 B=2*PI *L1/T1
1330 O=2*PI *S3/T1
1340 A(0,N)=F
1350 Z1=2*PI *R1/T1
1360 Z2=2*PI *S1/T1
1370 V2=-(3*G9*COS (-Z1+Z2-2*O-2*B))-G9*COS (-Z1+Z2+2*O+2*B)+G9*COS (-Z1+Z2-2*B)
-G9*COS (-Z1+Z2+2*B)
1380 V2=V2+G9^2*COS (-Z1+Z2-2*O)-COS (-Z1+Z2+2*O)+(G9^2+3)*COS (-Z1+Z2)
1390 V3=-(3*G9*COS (2*O+2*B))+G9*COS (2*O-2*B)+(G9^2-3)*COS (2*O)-2*G9*COS (2*B)
+5+G9^2
1400 A(1,N)=(COS (-Z1+Z2)-V2/V3*2)*V1
1410 RETURN
1420 ! THIS IS SIMPLE DELAY LINE
1440 DISP "INPUT DB/CM ATTN"
1450 INPUT D3
1460 DISP "INPUT DELAY LINE LENGTH"
1470 INPUT L1
1480 D1=D3*L1
1490 G9=10^(-(D1/10))
1500 GOSUB 1580
1510 RETURN
1520 B=2*PI *L1/T1
1530 Z1=2*PI *R1/T1
1540 Z2=2*PI *S1/T1
1550 A(0,N)=F
1560 A(1,N)=G9*COS (Z2-Z1+2*B)*V1
1570 RETURN
1580 F6=F1
1590 F7=F2-I1
1600 I=I1
1610 N1=0
1620 N=0
1630 FOR F=F6 TO F7 STEP I
1640 N=N+1
1650 T1=30000000000/(F*1000000000)/1.5
1660 ON C GOSUB 1310,1520
1670 P0=(10^(N9/10)-1)*10^(K2/10)*.001
1680 E2=(E1+SQR (P0*50))*10^(G1/20)
1690 IF F>F2 AND F<F2+.00007 THEN E2=(SQR (K1/((F-F2)*1000000000))+SQR (P0*50))*
10^(G1/20)
1700 IF N>2 THEN S4=(A(1,N)-A(1,N-1))/((A(0,N)-A(0,N-1))*1000000000)*10^(G1/20)
1710 IF F=F2 THEN N1=N
1720 IF N>2 THEN A(2,N-N1)=E2/S4
1730 NEXT F
1740 IF F>= F2+.00001 AND F<F2+.00007 THEN 1785
1745 IF F<F2+.00007 THEN 1785
1750 F6=F
1760 F7=F+.00001
1770 I=.0000001
1780 IF F<F3 THEN 1630
1785 IF F>F2+.00007 AND F<F3 THEN 1810
1786 F6=F
1787 F7=F+.00006
1788 I=.000001
1789 IF F<F3 THEN 1630
1790 T=N
1800 RETURN
1810 F6=F
1820 F7=F3
1830 I=I1
1840 GOTO 1630
```

LIST 1 BASIC CODE FOR DISCRIMINATOR SIMULATION WITH THE H.P. 87